

S-WAVE SCATTERING OF FERMION REVISITED

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A model where a Dirac fermion is coupled to background dilaton field is considered to study s-wave scattering of fermion by a back ground dilaton black hole. It is found that an uncomfortable situation towards information loss scenario arises when one loop correction gets involved during bosonization.

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I. INTRODUCTION

Scattering of fermion off dilaton black hole has been extensively studied over the years [1–3, 5–11] and it has provided much insight into its connection to the Hawking radiation. Even after the intensive investigations it remains as a subject of several interests because of the subtleties involved in it in connection with information loss scenario during the formation and subsequent evaporation of black hole. It is worth mentioning at this stage that a controversy in this context was generated from the Hawking's suggestion [1] three decades ago. However his recent suggestion on this issue [4] has brought back a pretty pleasant scenario. It may even be thought that the controversy has come to an end.

General description of such scattering problem is extremely difficult. Despite that, there have been attempts in studying such problem in its full complexity through the s-matrix description of such event [2]. Comparatively less complicated model therefore entered into the picture and showed its prominent role in this issue [5–9]. The toy model due to Alford and Strominger [5] provides an interesting description of the s-wave scattering of fermion off dilaton black hole with a trustworthy results concerning information loss. It helps to avoid some of the technical obstacle posed by quantum gravity in $(3+1)$. Even in the presence of gravitational anomaly a systematic description of this scattering of fermion off dilaton would have been possible through this model [10, 11]. The provision of taking the effect of one loop correction [12, 13] into consideration is also an exiting aspect of this model. That indeed shows a way to investigate the effect of anomaly [10, 11] on this scattering phenomena. Notably, this model arose in two dimensional non-critical string theory and its black hole solution was discovered in [14].

Few years back Mitra studied this scattering problem replacing Dirac fermion by chiral fermion and found an uncomfortable scenario [10]. He observed that information failed to be preserved. With the use of anomaly we have shown that that disaster can be avoided [11]. Anomaly played there a very surprising as well as interesting role. Seeing the interesting role of anomaly on the s-wave scattering of chiral fermion [10, 11] we are intended here to investigate the role of one loop correction on the s-wave scattering of Dirac fermion using the said toy model due to Alford and Strominger [5]. Needless to mention that the counter term appeared here due to one loop correction looks similar to the term used in [11].

II. TWO DIMENSIONAL EFFECTIVE MODEL FOR STUDYING S-WAVE SCATTERING

Here we consider only a special case which turns out to be particularly simple scattering of s-wave fermion incident on dilaton black hole. Black hole is the extrema of the following $(3+1)$ dimensional action.

$$S_{AF} = \int d^4x \sqrt{-g} [R + 4(\nabla\phi)^2 - \frac{1}{2}F^2 + i\bar{\psi}D\psi]. \quad (1)$$

Here g represents determinant of the space time metric. The geometry consists of three regions [5]. Far from the black hole there is an asymptotically flat region. The mouth leads to an infinitely long throat region. In side the throat region the metric is approximated by the flat metric on two dimensional Minkowsky space times the round metric on two sphere with radius Q . Electromagnetic field strength is tangential over the

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two sphere and an integer to $4\pi Q$. When the energy scales is large compared to Q , the radius of the two sphere the dynamics within the throat region can be described by the effective action

$$S_{AF} = \int d^2\sigma \sqrt{-g} [R + 4(\nabla\phi)^2 + \frac{1}{Q^2} - \frac{1}{2}F^2 + i\bar{\psi}D\psi], \quad (2)$$

Here $D_\mu = \partial_\mu + eA_\mu$. It is a two dimensional effective field theory of dilaton gravity coupled to fermion. Φ represents the scalar dilaton field and ψ is the charged fermion. For sufficiently low energy incoming fermion, gravitational effect on the scattering of s-wave fermion incident on a charge dilaton black hole can be neglected and equation (2) can be approximated by

$$S_f = \int d^2x [i\bar{\psi}\gamma^\mu [\partial_\mu + ieA_\mu]\psi - \frac{1}{4}e^{-2\Phi(x)}F_{\mu\nu}F^{\mu\nu}]. \quad (3)$$

The coupling e has one mass dimension. The indices μ and ν takes the values 0 and 1 in $(1+1)$ dimensional space time. The dilaton field Φ stands as a non dynamical back ground. It completes its role here just by making the coupling constant a position dependent function. This very toy model of quantum gravity in $(1+1)$ dimension contains black holes and Hawking radiation in a significant manner. Let us now define $G^2(x) = e^{2\Phi(x)}$. We will choose a particular dilaton background motivated by the linear dilaton vacuum of $(1+1)$ dimensional gravity like the other standard cases [5–11]. Therefore, $\Phi(x) = -x^1$, where x^1 is space like coordinate. The region $x^1 \rightarrow +\infty$, corresponds to exterior space where the coupling $G^2(x)$ vanishes and the fermion will be able to propagate freely. However, the region where $x^1 \rightarrow -\infty$, the coupling constant will diverge and it is analogous to infinite throat in the interior of certain magnetically charged black hole.

Equation (2), was derived viewing the throat region of a four dimensional dilaton black hole as a compactification from four to two dimension [3, 5, 7]. Note that, in the extremal limit, the geometry is completely non-singular and there is no horizon but when a low energy particle is thrown into the non-singular extremal black hole, it produces a singularity and an event horizon. The geometry of the four dimensional dilaton black hole consists of three significant regions [3, 5–8] as has already been mentioned. As long as one proceed nearer to the black hole the curvature begins to rise and finally enters into the mouth region (the entry region to the throat). In the deep throat region physics will be governed by the equation (2) since the metric at that region gets simplified into flat two dimensional Minkowsky metric times the round metric on the two sphere with radius Q . The dilaton field Φ indeed increases linearly with the proper distance into the throat.

We are now in a state to start our analysis and we would like to proceed with the bosonized version of the theory (3). During the process of Bosonization a one loop correction automatically enters within the action because bosonization needs to integrate out both the the left handed as well as the right handed part of the fermion one by one that leads to a fermionic determinant [12, 15, 16]. When this fermionic determinant is expressed in terms of scalar field a one loop correction enters into the theory in order to remove the divergence of the fermionic determinant. So the tree level bosonized theory gets the effect of loop correction during the process. Of course, bosonization can be done keeping the gauge symmetry intact which was used in [5]. Here masslike term for gauge field has been taken into consideration since we are intended to study the effect of this one loop correction in the s-wave scattering of Dirac fermion. With the counter term used in the study of non confining Scgwinner model [15, 16] the bosonized action reads

$$\mathcal{L}_B = \frac{1}{2}\partial_\mu\phi\partial^\mu\phi - e\tilde{\partial}_\mu\phi A^\mu + \frac{1}{2}ae^2A_\mu A^\mu - \frac{1}{4}e^{2\Phi(x)}F_{\mu\nu}F^{\mu\nu}. \quad (4)$$

Here ϕ represents a scalar field and $\tilde{\partial}_\mu$ is the dual to ∂_μ . $\tilde{\partial}_\mu$ is defined by $\tilde{\partial}_\mu = \epsilon_{\mu\nu}\partial^\nu$. Note that the lagrangian (4), maps onto the non-confining Schwinger model [15, 16] for $\Phi(x) = 0$.

The $U(1)$ current in this situation is

$$J_\mu = -e\epsilon_{\mu\nu}\partial^\nu\phi + ae^2A_\mu \quad (5)$$

and it is non conserving since $\partial_\mu J^\mu \neq 0$. This current was of preserving nature in [5–8] and in those situations the currents were $J_\mu = -e\epsilon_{\mu\nu}\partial^\nu\phi$. The new setting considered here indeed to show the role of the one loop correction on the s-wave scattering of Dirac fermion.

III. HAMILTONIAN ANALYSIS OF THE MODEL

It is now necessary to carry out the Hamiltonian analysis of the theory to observe the effect of the dilaton field on the equations of motion. From the standard definition the canonical momenta corresponding to

the scalar field ϕ , and the gauge fields A_0 and A_1 are found out:

$$\pi_\phi = \phi' - eA_1 \quad (6)$$

$$\pi_0 = 0, \quad (7)$$

$$\pi_1 = e^{-2\phi(x)}(\dot{A}_1 - A'_0) = \frac{1}{G^2}(\dot{A}_1 - A'_0). \quad (8)$$

Here π_ϕ , π_0 and π_1 are the momenta corresponding to the field ϕ , A_0 and A_1 . Using the above equations, it is straightforward to obtain the canonical hamiltonian through a Legendre transformation. The canonical hamiltonian is found out to be

$$\begin{aligned} \mathcal{H} = & \frac{1}{2}(\pi_\phi + eA_1)^2 + \frac{1}{2}e^{2\Phi(x)}\pi_1^2 + \frac{1}{2}\phi'^2 + \pi_1 A'_0 - eA_0\phi' \\ & - \frac{1}{2}ae^2(A_0^2 - A_1^2). \end{aligned} \quad (9)$$

Note that there is an explicit space dependence in the hamiltonian (9) through the dilaton field $\Phi(x)$ but it does not pose any hindrance to be preserved in time. So consistency and physically sensibility are in no way be threatened. Equation (7) is the familiar primary constraints of the theory. Therefore, it is necessary to write down an effective hamiltonian:

$$\mathcal{H}_{eff} = \mathcal{H}_C + u\pi_0 \quad (10)$$

where u is an arbitrary Lagrange multipliers. The primary constraints (7) has to be preserve in order to have a consistent theory. The preservation of the constraint (7), leads to the Gauss law of the theory as a secondary constraint:

$$G = \pi'_1 + e\phi' + ae^2A_0 \approx 0. \quad (11)$$

The preservation of the constraint (11) though does not give rise to any new constraint it fixes the velocity u which comes out to be

$$u = A'_1. \quad (12)$$

We, therefore, find that the phase space of the theory contains the following two second class constraints.

$$\omega_1 = \pi_0 \approx 0, \quad (13)$$

$$\omega_2 = \pi'_1 + e\phi' + ae^2A_0 \approx 0. \quad (14)$$

Both the constraints (13) and (14) are weak conditions up to this stage. When we impose these constraints strongly into the canonical hamiltonian (9), the canonical hamiltonian gets simplified into the following form.

$$\begin{aligned} \mathcal{H}_{red} = & \frac{1}{2}(\pi_\phi + eA_1)^2 + \frac{1}{2ae^2}(\pi'_1 + e\phi')^2 + \frac{1}{2}e^{2\Phi(x)}\pi_1^2 \\ & + \frac{1}{2}\phi'^2 + \frac{1}{2}ae^2A_1^2. \end{aligned} \quad (15)$$

\mathcal{H}_{red} obtained in equation (15), is generally known as reduced Hamiltonian. According to Dirac [17], Poisson bracket gets invalidate for this reduced Hamiltonian. This reduced Hamiltonian however remains consistent with the Dirac bracket which is defined by

$$\begin{aligned} [A(x), B(y)]^* &= [A(x), B(y)] \\ &- \int [A(x), \omega_i(\eta)] C_{ij}^{-1}(\eta, z) [\omega_j(z), B(y)] d\eta dz, \end{aligned} \quad (16)$$

where $C_{ij}^{-1}(x, y)$ is given by

$$\int C_{ij}^{-1}(x, z) [\omega_j(z), \omega_k(y)] dz = \delta(x - y) \delta_{ik}. \quad (17)$$

For the theory under consideration $C_{ij}(x, y) =$

$$ae^2 \begin{pmatrix} 0 & -\delta(x-y) \\ \delta(x-y) & o \end{pmatrix}. \quad (18)$$

Here i and j runs from 1 to 2 and ω 's represent the constraints of the theory. With the definition (16), we can compute the Dirac brackets between the fields describing the reduced Hamiltonian H_{red} . The Dirac brackets between the fields A_1 , π_1 , ϕ and π_ϕ are required to obtain the theoretical spectra (equations of motion):

$$[A_1(x), A_1(y)]^* = 0 = [\pi_1(x), \pi_1(y)]^*, \quad (19)$$

$$[A_1(x), \pi_1(y)]^* = \delta(x-y), \quad (20)$$

$$[\phi(x), \phi(y)]^* = 0 = [\pi_\phi(x), \pi_\phi(y)]^*, \quad (21)$$

$$[\phi(x), \pi_\phi(y)]^* = \delta(x-y). \quad (22)$$

The Dirac Brackets (19), (20), (21) and (22), along with the Heisenberg's equation of motion leads to the following four first order equations.

$$\dot{A}_1 = e^{2\Phi} \pi_1 - \frac{1}{ae^2} (\pi_1'' + e\phi''), \quad (23)$$

$$\dot{\phi} = \pi_\phi + eA_1, \quad (24)$$

$$\pi_\phi = \frac{a+1}{a} \phi'' + \frac{1}{ae} \pi_1'', \quad (25)$$

$$\dot{\pi}_1 = -e\pi_\phi - (a+1)e^2 A_1. \quad (26)$$

A little algebra converts the four first order equations into the following two second order Klein-Gordon equations:

$$[\square + (1+a)e^2 e^{2\Phi(x)}] \pi_1 = 0, \quad (27)$$

$$\square[\pi_1 + e(1+a)\phi] = 0. \quad (28)$$

The equation (27), represents a massive boson with square of the mass $m^2 = (1+a)e^2 e^{2\Phi(x)}$. Here a must be greater than -1 in order to have the mass of the boson a physical one. Equation (28), however describes a massless boson. The presence of this massless boson has a disastrous role here which will be going to uncovered now.

Let us concentrate into the theoretical spectra. Equation (27) shows that the mass of the boson is not constant in this model. It contains a position dependent factor $G^2 = e^{2\Phi(x)}$, where $\Phi = -x^1$, for the background generated by the linear dilaton vacuum of $(1+1)$ dimensional gravity. Therefore, $m^2 \rightarrow +\infty$ when $x^1 \rightarrow -\infty$ and $m^2 \rightarrow 0$ when $x^1 \rightarrow +\infty$. Thus mass of the boson goes on increasing indefinitely in the negative x^1 direction which implies that any finite energy contribution must be totally reflected and an observer at $x^1 \rightarrow \infty$ will get back all the information. To be more precise, mass will vanish near the mouth (the entry region to the throat) but increases indefinitely as one goes into the throat because of the variation of this space dependent factor G^2 . Since massless scalar is equivalent to massless fermion in $(1+1)$ dimension, we can conclude that a massless fermion proceeding into the black hole will not be able to travel an arbitrarily long distance and will be reflected back with a unit probability and a unitary s-matrix can be constructed. So there is no threat regarding information loss from the massive sector of the theory.

Of course it is a pleasant scenario. However an uncomfortable situation appears when we observe carefully towards the massless sector of the theory (28). It will remain massless irrespective of its position because unlike the massive sector it does not contain any space dependent factor. So this fermion will be able to travel within the black hole without any hindrance and an observer at $x^1 \rightarrow \infty$ will never find this fermion with a backward journey. Thus a real problem towards information loss appears for this setting. Note that in the similar type of studies, [5–8] where the setting was such that the masslike term for gauge field was absent, this problem did not occur. The result of the present work though leads to an uncomfortable situation, there is no known way to avoid it. It is true that after the Hawking’s recent suggestion [4] it seems to be an unwanted and untrustworthy scenario but one can not rule it out too if he has to accept the model [5]. More serious investigation is needed indeed in this issue. It is true that this result indicates a less brighter side of the model but it’s presence can not be ignored or suppressed.

IV. CONCLUSION

In this letter the s-wave scattering of fermion off dilaton black hole is investigated in presence of one loop correction due to bosonization. It was found that the presence of that correction term brings a disastrous result. Information loss could not be avoided. The result was not in agreement with the Hawking’s recent suggestion too. But there is no way to rule out this possibility.

Role of this type of quantum correction due to bosonization does not come as a great surprise for the first time. The crucial role of that was noticed earlier in the description of quantum electrodynamics and quantum chiral electrodynamics [12, 15, 16, 18–22]. A famous instance in this context is the removal of the long suffering of the chiral electrodynamics from the non unitary problem [12].

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